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String theoretic bounds on Lorentz-violating warped compactification

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Abstract: We consider warped compactifications that solve the 10 dimensional supergravity equations of motion at a point, stabilize the position of a D3-brane world, and admit a warp factor that violates Lorentz invariance along the brane. This gives a string embedding of "asymmetrically warped" models which we use to calculate stringy (α') corrections to standard model dispersion relations, paying attention to the maximum speeds for different particles. We find, from the dispersion relations, limits on gravitational Lorentz violation in these models, improving on current limits on the speed of graviton propagation, including those derived from field theoretic loops. We comment on the viability of models that use asymmetric warping for self-tuning of the brane cosmological constant.

Keywords: Extra Large Dimensions, D-branes, Space-Time Symmetries.

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Contents

1. Introduction

Violation of Lorentz invariance is of great interest in the field of quantum gravity since it can arise in many different ways [1]. For example, canonical quantum gravity has been argued to predict Planck scale Lorentz violation [2], and noncommutative space-time coordinates break Lorentz invariance [3]. Lorentz violation has also been related to variation of coupling constants [4]. The literature is very broad, and we refer the reader to [1] for a review of several developments.

In this paper, we consider a form of Lorentz violation that can appear in braneworld compactifications of string theory, which is known as asymmetric warping. In braneworld compactifications, the spacetime manifold need not be a direct product of noncompact and compact submanifolds; the noncompact spacetime metric usually has a warp factor dependent on the internal space (see [5, 6, 7] for early work on warping and a review). However, nothing in the higher dimensional theory keeps the warp factors of the different noncompact dimensions the same, as first discussed in [8]. The metric takes the form

$$ds^2 = -e^{A(y)}dt^2 + e^{B(y)}d\vec{x}^2 + \tilde{g}_{mn}(y)dy^m dy^n \,. \tag{1.1}$$

A nice feature of this form of Lorentz violation is that it is classically confined to the

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gravitational sector (the graviton moves at a different speed than light); if the brane stays at a fixed position in the extra dimensions, we can rescale the noncompact coordinates and recover an $\mathrm{SO}(3,1)$ symmetry for fields confined to the brane, including the standard model (in most braneworlds). These models therefore avoid, at tree level, the rather stringent limits on Lorentz violation in the standard model (see [9] for a recent review and section 4 for a brief discussion of the experimental bounds).

Despite the classical restriction of Lorentz violation to gravity, we expect that a symmetry broken anywhere is broken everywhere. Indeed, the communication of Lorentz violation from gravity to the standard model by quantum loops was studied in [10] in a rather general framework. Limits on Lorentz violation in the standard model are thus related to limits on asymmetric warping, which [10] quoted in terms of the difference of graviton and photon speeds. Evading limits such as these is a new problem for extra-dimensional cosmology, similar to the original flatness problem that was solved by inflation [11].

In parallel with [10], we consider the communication of Lorentz violation from gravity to the standard model sector in a string theoretical background, which we describe in section 2. Instead of loop corrections, we consider α' corrections to the D3-brane action, which are at tree level in string perturbation theory. Since the worldsheet disk fluctuates in the dimensions orthogonal to the D-brane, these terms are sensitive to the Lorentz violating derivatives of the metric. (Using stringy corrections has the slight advantage over loops that we do not have to worry about how to regulate perturbative gravity.) We calculate and describe the Lorentz violating contributions (from geometry only) to standard model propagators in section 3. Because the α' corrections we calculate involve only the metric, they do not give all possible terms, but they give estimates that are independent of the background of the other supergravity fields. In section 4, we then use our results to put limits on Lorentz violation in Randall-Sundrum [5, 6] type braneworlds and comment on asymmetric warping in self-tuning braneworlds [12, 13]. Our limits imply that the energy density that is self-tuned cannot be within a few orders of magnitude of the higher dimensional fundamental scale, so there must be some other mechanism to explain the smallness of the brane energy density. We close with a short discussion in section 5. We do not address a solution to the flatness problem of [11], but our results do sharpen it somewhat.

2. A simple string braneworld

In this section, we describe a braneworld model that solves the 10D type-IIB supergravity equations of motion locally at the position of the brane. We treat the brane as a string theoretic D3-brane in the approximation that it probes the ambient geometry without affecting it. Since we are interested in the effect of the geometry on the D3-brane fields, we justify the probe approximation by noting that a static brane should not be affected by its own action on the background (as in electrostatics). We will choose a background that stabilizes the position of the D3-brane and allows for asymmetrical warping. We are most interested in making order of magnitude estimates for Lorentz-breaking effects, so we will not try to incorporate the standard model.

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For simplicity, we take the type-IIB 3-form field strengths and RR scalar to vanish and dilaton to be constant with string coupling g_s. The equations of motion for the 5-form field strength and the string frame metric then become (see [14], for example, using the conventions of [15])

$$R_{MN} = \frac{g_s^2}{96}\tilde{F}_{MPQRS}\tilde{F}_N{}^{PQRS} \tag{2.1}$$
$$d\tilde{F} = 0\,, \qquad \star\tilde{F} = \tilde{F}\,. \tag{2.2}$$

We want to confine Lorentz violation to the gravitational sector, so we can take an ansatz[1]

$$\begin{aligned} ds^2 &= -e^{A(y)}dt^2 + e^{B(y)}d\vec{x}^2 + \tilde{g}_{mn}(y)dy^m dy^n \\ \tilde{F}_{\mu\nu\lambda\rho m} &= -\frac{1}{g_s}\epsilon_{\mu\nu\lambda\rho}\partial_m F(y)\,, \qquad \tilde{F}_{mnpqr} = \frac{1}{g_s}\epsilon_{mnpqr}{}^s\partial_s F(y)\,, \end{aligned} \tag{2.3}$$

where ϵ is the volume form on spacetime or the internal manifold respectively. Note that this ansatz automatically satisfies the 5-form equations (2.2). At the position of the D3-brane, $y^m = 0$, the y^m are Riemann normal on the internal manifold[2] ($\tilde{g}_{mn}(y=0) = \delta_{mn}$, $\partial_p\tilde{g}_{mn}(y=0) = 0$), and the functions A, B, F have Taylor expansions $A(y) = a_m y^m + (1/2)a_{mn}y^m y^n + \cdots$, etc.

At zeroth order in y^m, the Einstein equation (2.1) becomes

$$\begin{aligned} R_{00} &= \frac{1}{2}\left(a_m a^m + a_m{}^m\right) = \frac{1}{4}f_m f^m \\ R_{ij} &= \frac{1}{2}\left(b_m b^m + b_m{}^m\right)\delta_{ij} = -\frac{1}{4}f_m f^m \delta_{ij} \end{aligned} \tag{2.4}$$

along the D3-brane and

$$R_{mn} = \tilde{R}_{mn} + \frac{1}{2}a_{mn} - \frac{3}{2}b_{mn} = -\frac{1}{4}\left(2f_m f_n - f_p f^p \delta_{mn}\right) \tag{2.5}$$

transverse to it, where $\tilde{R}_{mn}$ is the Ricci tensor for the internal metric $\tilde{g}_{mn}(y)$. The other components are trivial. Note that eq. (2.5) is extraneous, since we can satisfy it by choosing manifolds with the appropriate $\tilde{R}_{mn}$ (for example, many cases are satisfied with a warped Calabi-Yau manifold). Since we will also want the second derivative of the function F later, we should look at the next order in y^m; along the brane, we have

$$\begin{aligned} \left(a_n a^n + a_n{}^n\right)a_m + 2a^n a_{nm} &= f^n f_{mn} \\ \left(b_n b^n + b_n{}^n\right)b_m + 2b^n b_{nm} &= f^n f_{mn}\,. \end{aligned} \tag{2.6}$$

To work within perturbation theory, we should stabilize the D-brane position. From the action for D-branes (see, for reviews, [15, 16]), the potential for the brane scalars —

[1] We take Greek indices on spacetime, lower case Roman from the middle of the alphabet on the internal manifold, and capital Roman for all 10 dimensions. We will usually use vector notation for spatial directions of the noncompact spacetime but will sometimes denote them with indices i, j, k. Hats will indicate orthonormal basis indices, as for a tangent space.

[2] Through the rest of this paper, we use "Riemann normal coordinates" to refer to coordinates that are Riemann normal on the 6D internal manifold, ignoring the rest of the metric.

the brane coordinates — is the determinant of the pulled-back metric minus the 4-form potential,

$$V = \frac{\tau}{g_s} e^{(A+3B)/2} - \tau C_{0123} \tag{2.7}$$

with brane tension τ/g_s. For the forces on the brane to balance, we require

$$\frac{1}{2}a_m + \frac{3}{2}b_m + f_m = 0 \,. \tag{2.8}$$

The scalar masses come from the second derivative of the potential,

$$\frac{2g_s}{\tau}\partial_m\partial_n V = \frac{1}{2}\left(a_m + 3b_m\right)\left(a_n + 3b_n\right) + a_{mn} + 3b_{mn} + 2g_s f_{mn} \,. \tag{2.9}$$

To stabilize all the brane coordinates, the trace of this matrix must be positive. Using eqs. (2.4), (2.8), we get

$$\frac{2g_s}{\tau}\partial^2 V = f_m f^m + 2g_s f_m{}^m - a_m a^m - 3b_m b^m \,. \tag{2.10}$$

The equations of motion (2.6) seem not to constrain f_{mn} tightly, so there seems no obstacle to stabilizing the brane position. We should note, however, that without the 5-form, the potential is at best flat or is at a saddle point. (In fact, with $a_m = b_m$, we get just the $f_m{}^m$ term.)

We close this section by noting that this is by no means the most general way to stabilize the brane position; for example, a nontrivial dilaton and 3-form flux can contribute to the scalar potenial for multiple D-branes [17, 16] ([18] gives a detailed mapping of the SUGRA fields to MSSM parameters in a particular model). However, since our purpose is to demonstrate that α' corrections communicate Lorentz breaking from the pure gravity sector to the brane fields, our background is sufficient. From this point forward, we need only refer to the geometrical part of our background (2.3).

3. Stringy tree level corrections

In this section, we will identify terms in the action for brane fields that violate Lorentz invariance due to worldsheet interactions with the background metric. We begin by examining the zero-th order action for all the brane fields and verifying that there are no violations of Lorentz invariance. Then we move to leading order α' corrections for the scalars and gauge fields. Finally, we comment on corrections we do not calculate but which should appear. Through all the analysis, we look for terms at second order in the brane fields because most experimental limits come from Lorentz violating dispersion relations. The strongest limits are on different effective "speeds of light" (more appropriately, maximum attainable velocities, or MAVs) for different fields, so we will calculate just the difference of MAVs from one for various D-brane fields.

3.1 κ symmetric D-brane action

Including fermions, the action for a D3-brane [19, 20] is

$$S = -\tau \int d^4\zeta\, e^{-\boldsymbol{\Phi}} \sqrt{-\det\left(\boldsymbol{g}_{\alpha\beta} + \boldsymbol{\mathcal{F}}_{\alpha\beta}\right)} + \tau \int e^{\boldsymbol{\mathcal{F}}} \wedge \boldsymbol{C}\,. \tag{3.1}$$

The fields in boldface are superfields in type-II 10D superspace, where

$$\boldsymbol{\mathcal{F}} = 2\pi\alpha' F - \boldsymbol{B}\,, \qquad \text{and} \qquad \boldsymbol{C} = \oplus_n \boldsymbol{C}_{(n)} \tag{3.2}$$

is the collection of the pullbacks of the RR potentials. Here, Greek letters from the beginning of the alphabet refer to directions tangent to the D3-brane, which are to lowest order in the brane fluctuations $\zeta^\alpha = \delta^\alpha_\mu x^\mu$. There is a local κ symmetry on the worldvolume that reduces the Grassman superspace coordinates to a single $\mathrm{SO}(9,1)$ Majorana-Weyl spinor Θ that contains four worldvolume $\mathrm{SO}(3,1)$ fermions.

The expansion of this action in terms of the D-brane scalars Y^m, gauge field $F_{\alpha\beta}$, and fermions Θ was worked out in [21] for a background similar to ours (slightly more general but Lorentz invariant). It is straightforward to write down the renormalizable part of the action for our background (with the flat metric at $y^m = 0$):

$$\begin{aligned} S = -\frac{\tau}{g_s}\int d^4x \Big[&\frac{1}{2}\delta_{mn}\partial_\mu Y^m \partial^\mu Y^n + \frac{g_s}{\tau}V(Y) + \frac{(2\pi\alpha')^2}{4}F_{\mu\nu}F^{\mu\nu} - \\ &- i\overline{\Theta}\Gamma^\mu\partial_\mu\Theta + \frac{i}{4}\omega_{\mu\hat{M}\hat{N}}\overline{\Theta}\Gamma^\mu\Gamma^{\hat{M}\hat{N}}\Theta\Big]\,. \end{aligned} \tag{3.3}$$

The tension is given by[3] $\tau = (2\pi)^{-1}(2\pi\alpha')^{-2}$ and the potential should only be taken out to quartic order in Y^m (there are additional terms at higher order in Y, Θ if we include nonrenormalizable interactions). The last term includes $\omega_{\mu\hat{M}\hat{N}}$, the 10D spin connection 1-form in the direction along the brane. The only vanishing components are

$$\omega_0{}^{\hat{0}\hat{m}} = -\frac{1}{2}a_m e^{m\hat{m}}\,, \qquad \omega_i^{\hat{j}\hat{m}} = -\frac{1}{2}b_m e^{m\hat{m}}\delta_i^{\hat{j}}\,, \tag{3.4}$$

so the final term becomes

$$-\frac{i}{8}\left(a_m + 3b_m\right)\overline{\Theta}\Gamma^m\Theta\,. \tag{3.5}$$

It has been shown in [21] that this linear combination of spinors vanishes due to the Majorana-Weyl nature of Θ. At $Y^m = 0$, this is the only term that could have communicated the Lorentz breaking from the gravity sector to the brane fields, so we must turn to stringy corrections.[4]

[3]This gives a gauge coupling of $g_{\mathrm{YM}}^2 = g_s$. Since we are taking a single D3, we ignore terms associated with nonabelian gauge theory.

[4]Clearly, there are Lorentz breaking terms in the nonrenormalizable part, since the metric $g_{\mu\nu}$ is not Lorentz invariant at $y^m \neq 0$. We want instead changes to dispersion relations, since limits on those are known.

3.2 $(\alpha')^2$ corrections to dispersion relations

Even at tree level in string amplitudes, the action (3.1) is incomplete; integrating out higher string modes gives a series of higher derivative terms with α' as the expansion parameter. We see below that, when some of the derivatives act on the background, these corrections can actually modify the propagator (in the language of loops, renormalize the wavefunction). Next, we will review the $\mathcal{O}(\alpha')^2$ corrections that we consider, along with the necessary geometry.

3.2.1 Corrected DBI action and embedding geometry

Probably the most famous α' corrections to D-brane actions are from the couplings to the RR potentials; Riemann curvature becomes lower-dimensional brane charge. The Wess-Zumino part of the action is

$$S_{WZ} = \tau \int C \wedge e^{\mathcal{F}} \wedge \sqrt{\frac{\hat{A}\left(4\pi^2\alpha' R_T\right)}{\hat{A}\left(4\pi^2\alpha' R_N\right)}} \tag{3.6}$$

where $\hat{A}$ is the Dirac "A-roof genus" and R_T, R_N are the tangent and normal bundle Riemann tensors to be defined below (see [22] for a full derivation and [16] for a review). Although these corrections are generally nonvanishing for our background, we are not interested in them. We do note that they modify the background, since now the D3-brane will have charge under the RR scalar, but this is a subleading effect and will not affect our conclusions.

To get the modified dispersion relations, we need to consider the corrections to the DBI action, which have been studied in [23]–[27]. The most complete results for curved backgrounds are in [25], but only the geometry is considered and not other SUGRA fields. The DBI action becomes (considering the bosonic part only)

$$\begin{aligned} S_{\rm DBI} = & -\tau \int d^4\zeta\, e^{-\Phi} \sqrt{-\det\left(g_{\alpha\beta}+\mathcal{F}_{\alpha\beta}\right)} \times \\ & \times \left[1 - \frac{(2\pi\alpha')^2}{192}\left((R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta} - 2(R_T)_{\alpha\beta}(R_T)^{\alpha\beta} - \right.\right. \\ & \left.\left. - (R_N)_{\alpha\beta\hat{a}\hat{b}}(R_N)^{\alpha\beta\hat{a}\hat{b}} + 2\bar{R}_{\hat{a}\hat{b}}\bar{R}^{\hat{a}\hat{b}}\right)\right] \end{aligned} \tag{3.7}$$

up to $\mathcal{O}(\alpha')^2$. There is an additional contribution at this order with an undetermined coefficient, but it vanishes on-shell, so it does not affect S-matrix elements or dispersion relations [25]. Therefore, we can ignore it. Here, $\hat{a}, \hat{b}$ are normal bundle indices in an orthonormal basis with vielbein $\xi^{\hat{a}}$.

We now define the various Riemann and Ricci tensors above, as is discussed in [23]–[26]. In the following, we use $P[\cdots]$ to denote the pullback to the worldvolume or pushforward to the normal bundle, but for brevity we write $g_{\alpha\beta} \equiv P[g]_{\alpha\beta}$ and $g^{\alpha\beta} \equiv \left(P[g]_{\alpha\beta}\right)^{-1}$. Start with the extrinsic curvature, or second fundamental form, of the D-brane embedding

$$\Omega^{\hat{a}}_{\alpha\beta} = \xi^{\hat{a}}_M \left(\partial_\alpha\partial_\beta X^M - (\Gamma_T)^\gamma_{\alpha\beta}\partial_\gamma X^M + \Gamma^M_{NP}\partial_\alpha X^N \partial_\beta X^P\right), \tag{3.8}$$

where $(\Gamma_T)^\gamma_{\alpha\beta}$ is the Christoffel connection of the pulled-back metric. The tangent and normal bundle Riemann tensors can be shown to be

$$(R_T)_{\alpha\beta\gamma\delta} = P[R]_{\alpha\beta\gamma\delta} + \delta_{\hat{a}\hat{b}}\left(\Omega^{\hat{a}}_{\alpha\gamma}\Omega^{\hat{b}}_{\beta\delta} - \Omega^{\hat{a}}_{\alpha\delta}\Omega^{\hat{b}}_{\beta\gamma}\right), \tag{3.9}$$

$$(R_N)_{\alpha\beta}{}^{\hat{a}\hat{b}} = P[R]_{\alpha\beta}{}^{\hat{a}\hat{b}} + g^{\gamma\delta}\left(\Omega^{\hat{a}}_{\alpha\gamma}\Omega^{\hat{b}}_{\beta\delta} - \Omega^{\hat{b}}_{\alpha\gamma}\Omega^{\hat{a}}_{\beta\delta}\right). \tag{3.10}$$

Then $(R_T)_{\alpha\beta}$ is just the Ricci tensor associated with the tangent Riemann tensor, and

$$\bar{R}^{\hat{a}\hat{b}} = g^{\alpha\beta}P[R]_\alpha{}^{\hat{a}\hat{b}}{}_\beta + g^{\alpha\gamma}g^{\beta\delta}\Omega^{\hat{a}}_{\alpha\beta}\Omega^{\hat{b}}_{\gamma\delta}. \tag{3.11}$$

As we continue, we should remember that the action (3.7) assumes that all the other fields vanish. We will address this point in section 3.3, and we will below use T-duality to deduce possible couplings between the extrinsic curvature and the worldvolume field strength. Also, the terms involving fermions are not known, although [27, 28] give steps in that direction. We will consider that the corrections to fermions will be similar to those for the scalars, which we can calculate.

3.2.2 Maximum attainable velocities for scalars

To get the modified MAVs for the scalar fields on the brane, we just need to calculate the Lorentz violating kinetic terms that arise in the corrected action (3.7). We can write these all as $(\cdots)\vec{\partial}Y\cdot\vec{\partial}Y$ (because $\partial_0 Y\partial_0 Y$ differs by a Lorentz invariant term). Knowing this allows us to simplify our calculation greatly; we need keep only terms up to $\mathcal{O}(\partial Y)^2$ and can ignore terms in which Y appears without a derivative as well as $\partial^2 Y$ terms. Also, since the background would be Lorentz invariant if the warp factors were equal, $A = B$, we can write $A = B + \Lambda$ and take only terms in which Λ, the Lorentz violating function, appears. To get the best limits on Lorentz violation, we keep only terms that are linear in Λ.

The key to our results is that the extrinsic curvature has terms that are zero-th order in the brane fluctuations that come from $\Gamma^m_{\mu\nu}$. In our background, the extrinsic curvature is

$$\begin{aligned}
\Omega^m_{00} &= \frac{1}{2}a^m - a_n\partial_0 Y^n\partial_0 Y^m + \cdots,\\
\Omega^m_{0i} &= -\frac{1}{2}a_n\partial_i Y^n\partial_0 Y^m - \frac{1}{2}b_n\partial_0 Y^n\partial_i Y^m + \cdots,\\
\Omega^m_{ij} &= -\frac{1}{2}b^m\delta_{ij} - b_n\partial_{(i}Y^n\partial_{j)}Y^m + \cdots.
\end{aligned} \tag{3.12}$$

We use standard notation regarding symmetrization of indices with a weighting of 1/2. Also, to reduce proliferation of indices, we replace the normal bundle index $\hat{a}$ with the coordinate index m since they are the same at lowest order in perturbation theory. We have still included all the appropriate terms from the expansion of the normal bundle vielbein, however. Additional zero-th order terms appear in the Riemann tensor part of $\bar{R}^{mn}$. We leave those, along with the details of the rest of the calculation to the appendix A.

After much algebra, and carefully accounting for all terms, we find the following Lorentz violating kinetic terms for the scalars to linear order in Λ:

$$\delta S = \frac{\tau}{g_s}\int d^4x \frac{(2\pi\alpha')^2}{192}\vec{\partial}Y^m \cdot \vec{\partial}Y^n \left[\frac{21}{2}b^2 b_{(m}\lambda_{n)} + 5b\cdot\lambda b_m b_n - \frac{1}{2}b^2\lambda_{mn} + b\cdot\lambda b_{mn} - \right. \\ - 4\lambda^p b_{p(m}b_{n)} - 4b^p b_{p(m}\lambda_{n)} + \frac{47}{4}b^2 b\cdot\lambda\delta_{mn} + \\ \left. + 4b^{pq}\lambda_{pq}\delta_{mn} + 4\lambda^{pq}b_p b_q\delta_{mn} + 20b^{pq}\lambda_p b_q\delta_{mn}\right]. \quad (3.13)$$

Because the action (3.7) does not include contributions from the background 5-form, these are probably not the only Lorentz violating kinetic terms for the scalars. However, because Lorentz invariance is broken by the background, there is no reason to suppose that the terms we have calculated are precisely canceled by those that we have not. Therefore, we can take these as an estimate of the change in MAV for the scalars. From (3.13), we can see clearly why these terms modify the MAV of the scalars; by a *field dependent* redefinition of the "speed of light," we can clearly combine all the time and space derivatives of the fields into the usual form $\partial_\mu Y \partial^\mu Y$. Each scalar has a Lorentz invariant kinetic action, but the Lorentz groups are in general different for the different scalars. This way of thinking about our corrections is essentially the formalism of [29].

Of course, even at this order in the string expansion, there are many other corrections to the action. For example, the constant terms in the extrinsic curvature give a cosmological constant at when the brane is at $y^m = 0$; more generally, there is a new contribution to the potential for the D-brane position. We are not really expanding around the vacuum of the theory. Similarly, the supergravity background should be altered (even if we had incorporated the tension and charge of the brane to begin with) because the A-roof corrections induce a charge for other supergravity fields, notably the RR scalar. We will not worry about these effects, since they will be suppressed by $\alpha'^2 b^4$, and the warp factor should be somewhat less curved than the string scale for supergravity to be valid in the first place. Thus, in perturbation theory, they do not affect the leading Lorentz violation. Also, as we mentioned above, there will be other corrections due to the 5-form field strength, and we know that for the "black 3-brane" background the corrections should all cancel because of supersymmetry. As another example, the scalar wavefunctions will be renormalized by Lorentz invariant kinetic terms in the correction; again, this is suppressed by $\alpha'^2 b^4$. Finally, there are many, generally nonrenormalizable, interactions introduced.

3.2.3 Maximum attainable velocity for photon

To find the communication of Lorentz violation to electrodynamics from gravity, we proceed by a somewhat indirect route. The derivative corrections to the DBI action are not known in a curved background, so, rather than generalize the flat space results of [24, 26], we use T-duality to relate the field strength to the scalars. We will also argue that our results are plausible terms in the curved-space generalization of [24, 26].

Our T-duality argument runs as follows. Suppose we compactify one of the noncompact spatial dimensions, say x^3, on a large circle. Then we perform T-duality on x^3, giving a metric of

$$ds^2 = -e^A dt^2 + e^B \sum_{i=1}^{2} \left(dx^i\right)^2 + e^{-B}\left(dx^3\right)^2 + \tilde{g}_{mn} dy^m dy^n \tag{3.14}$$

with, as before, all components of the metric depending only on y^m. Now x^3 is compactified on a small circle, and the braneworld is a D2-brane. In fact, this metric is of the same form as our original metric (2.3) except that x^3 is not a Riemann normal coordinate. The only difference this makes from the calculations of section 3.2.2 is that we now have to keep in mind that there is a nonvanishing Christoffel symbol Γ^m_{33} that contributes kinetic terms to $\Omega^m_{\alpha\beta}$ (Γ^3_{3m} will not matter because $\Omega^3_{\alpha\beta}$ will already be second order in $\partial X, \partial Y$). Otherwise some numerical factors differ. If we recalculate eq. (3.13) for fluctuations of X^3 in 2 dimensions, we find

$$\delta S = \frac{\tau_2}{g_{s,2}} \int d^3x \frac{(2\pi\alpha')^2}{192} \left|\vec{\partial} X^3\right|^2 \left[\frac{11}{4} b^2 b \cdot \lambda + 3 b^{mn} \lambda_{mn} + 3 \lambda^{mn} b_m b_n + 9 b^{mn} \lambda_m b_n\right]. \tag{3.15}$$

Here $\tau_2, g_{s,2}$ are the appropriate tension and string coupling for the D2-brane case.

If we then T-dualize back to the D3-brane and take the x^3 circle radius to infinity, we get back the original tension and string coupling and take $X^3 \to (2\pi\alpha') A_3$. Then by gauge invariance, we must replace $(2\pi\alpha')\partial_i A_3 \to \mathcal{F}_{i3}$, and isotropy requires that we promote $\mathcal{F}_{i3}\mathcal{F}^{i3} \to \mathcal{F}_{ij}\mathcal{F}^{ij}$. Therefore, we end up with a correction to the photon kinetic term of

$$\delta S = \frac{\tau}{g_s} \int d^4x \frac{(2\pi\alpha')^2}{192} \mathcal{F}_{ij}\mathcal{F}^{ij} \left[\frac{11}{4} b^2 b \cdot \lambda + 3 b^{mn} \lambda_{mn} + 3 \lambda^{mn} b_m b_n + 9 b^{mn} \lambda_m b_n\right]. \tag{3.16}$$

So we see that even the speed of light is modified; the MAV of a photon is not 1!

Let us check this term against the known $\mathcal{O}(\alpha')^2$ corrections for the worldvolume gauge field. For a flat background, [24] showed that the corrections can be given in terms of a non-symmetric metric $h_{\alpha\beta} = g_{\alpha\beta} + \mathcal{F}_{\alpha\beta}$ and its Riemann tensor. So in curved space, we could easily have (Riemann,Ricci)$^2\mathcal{F}^2$ terms, including such terms as $\bar{R}_{mn}\Omega^m_{ik}\Omega^n_{jl}\mathcal{F}^{ij}\mathcal{F}^{kl}$. As it turns out, the method of replacing $g_{\alpha\beta} \to h_{\alpha\beta}$ only works to $\mathcal{O}(\alpha')^2$ for the Wess-Zumino terms when the background is curved, but we are working only to that order in any case.

3.3 Other corrections

We already discussed some of the other corrections that appear for scalars at the end of section 3.2.2. These are $\mathcal{O}(\alpha')^2$ terms that we could have calculated but have not (because the experimental limits are far greater on the dispersion relations that we have calculated). One additional correction of this type is of second order in the scalars but fourth order in derivatives; they introduce quartic terms in the dispersion relation. Some of them T-dualize to a modification for the field strength of the form $\vec{\partial}\mathcal{F}_{ij} \cdot \vec{\partial}\mathcal{F}^{ij}$, and there are actually experimental limits on such terms. However, they are very weak compared to the limits on the second order terms [9].

We are not just limited to $\mathcal{O}(\alpha')^2$ corrections, even though these are the only ones known (and incompletely, at that). Because the extrinsic curvature and some of the Riemann components have terms that are zero-th order in D-brane fields and their derivatives, we expect that all orders in α' should contribute to kinetic terms, with a suppression by the string length over the warp factor curvature length. This should be small for supergravity to remain valid, but it should still be within a few orders of magnitude of unity in a Randall-Sundrum type model with only one fundamental scale [5, 6]. Of interest is whether the Riemann tensor of the internal metric $\tilde{g}_{mn}$ can contribute to the Lorentz violating terms at some order in the α' expansion; in appendix A, we see that it couples to the scalars only in a Lorentz invariant way in the terms we consider. Since, in any realistic (say a warped Calabi-Yau) compactification, we expect the curvature of the compact manifold to be of order the string scale (and therefore the fundamental Planck scale), such terms could contribute to very high orders in α'.

We also mention again the fact that there should be corrections due to the other background bulk supergravity fields even at $\mathcal{O}(\alpha')^2$. Unless they also couple to the background metric, we would not expect them to violate Lorentz invariance as long as their background does not, but it would be interesting to be able to study them.

Most importantly, the worldvolume fermions should also have Lorentz violation kinetic terms from α' corrections to the κ symmetric D-brane action. Unfortunately, the $\mathcal{O}(\alpha')^2$ corrections to the DBI action are not known in κ symmetric form, so we cannot calculate them. Progress in this direction was reported in [28], but we will simply estimate that the Lorentz violating terms will be roughly the same for the fermions as for the scalars.

We close this section with a comment about the applicability of our results, then. We have calculated only the purely gravitational contribution to the Lorentz violating kinetic terms and only at lowest order in α'. It is very likely, at least in some models, that contributions from other fields or even from higher orders in α' (because the internal Riemann curvature should go as $1/\alpha'$) could also be significant. Therefore, the reader should not consider the limits we derive here to be definitive for any specific model (especially since the stringy details of most models are not yet resolved). Rather, the limits we get are somewhat rough but model independent because they depend only on the metric. Additionally, because the Lorentz invariance is broken by the background geometry, extra contributions to the α' corrections should not change the order of magnitude of our limits. Almost any model of asymmetric warping, if considered as a solution of string theory, will be subject to them.

4. Limits on asymmetric warping

We can now use the results of section 3 to apply known experimental limits on Lorentz violation in the standard model to asymmetric warping in the gravitational sector. In particular, many unobserved effects, such as Čerenkov radiation by charged particles in vacuum and photon decay, can occur if the MAVs of photons and fermions are different [29].[5] In fact, vacuum Čerenkov radiation and photon decay are so efficient that we should observe no particles above threshhold for those interactions (see, for example [30]). These two interactions give the constraint that the magnitude of the difference of photon

[5] We do not consider interactions, such as electromagnetic muon decay, that also violate lepton number.

and electron MAVs should not be greater than 10^{-16} (in units with the speed of light equal to unity). A demonstration of this result, along with a review of many dispersion relation tests, is given in [9].

In fact, we could use more stringent constraints. For example, ignoring the parton structure, the proton MAV should differ from the photon MAV by no more than 10^{-22} [9]. Atomic spectroscopy experiments measuring spatial anisotropy of nuclear dipole and quadrupole couplings give a similar (indirect) bound of 10^{-22} [31]; this is the limit used by [10]. Additionally, other Lorentz (and even CPT) violating effects in QED have been considered by many authors (see, for example, [32, 33]). However, these mainly deal with polarization effects, and we will not consider them. To be conservative, we will require that the difference in fermion and photon MAVs be less than 10^{-16} in absolute value. However, for comparison to the results of [10], we will also quote results using a limit of 10^{-22}.

From equations (3.13), (3.16), we can therefore see that, including the numerical factors, we should have

$$\alpha'^2 (b_m)^3 \lambda_m\,, \qquad \alpha'^2 b_{mn} \lambda_m b_m\,, \qquad \alpha'^2 \lambda_{mn} (b_m)^2\,, \qquad \alpha'^2 \lambda_{mn} b^{mn} < \epsilon \equiv 10^{-16}, 10^{-22} \tag{4.1}$$

in terms of magnitude. We will use this to put limits on the Lorentz violating function Λ in the following subsection, and we will relate this bound to the speed of graviton propagation in the asymmetrically warped background. Like [10], we will find the range of parameter space when our limits improve on the bound $|c_g - 1| < 10^{-6}$ (see [34] for a discussion relating this bound to experiments).

4.1 Perturbations around Randall-Sundrum

The prototypical warped braneworld models are those of Randall and Sundrum [5, 6] (see [7] for a general review), which have one or two branes in a total of 5 dimensions. Therefore, we take the asymmetric warping to be a small perturbation around the Lorentz invariant Randall-Sundrum metric below. To get our limits, we will assume that the other 5 dimensions of string theory are compactified at the 5-dimensional Planck scale M_5, so the fundamental 10D scale, the string scale, is the 5D scale, $\alpha' = M_5^{-2}$. It is trivial to extend our analysis to other cases given some specific model and our results should not be significantly changed; however, we will focus on the most basic case here.

If the visible sector brane is at $y = 0$, the Randall-Sundrum metric is

$$ds^2 = e^{\mp 2k|y|} dx^\mu dx_\mu + dy^2 \tag{4.2}$$

where the $-(+)$ sign corresponds to the one (two) brane model. This is AdS_5 with curvature given by the mass scale k. To make the supergravity approximation valid but to avoid naturalness problems, we should have $k \lesssim M_5$. For concreteness, we will quote limits taking the somewhat arbitrary value $k = M_5/10$.

We should also briefly address the warping in the other 5 dimensions, since they enter into the limits (4.1). In a traditional compactification, this warping vanishes, and, in more general warped compactifications, those warp factors should not be larger than the fundamental Planck scale if we can use a geometrical interpretation. A related issue is the origin of the 5D cosmological constant. In string theory, this would be related to the background supergravity fields, and, in the simple model of section 2, the 5-form plays the role of a negative cosmological constant (see the Einstein equations (2.4), (2.5)). As long as f_m is aimed along the Randall-Sundrum y direction, this is the same situation as in the $\mathrm{AdS}_5 \times S^5$ solution of string theory. Because of the equilibrium condition (2.8), we would not have large warp factors in the extra 5 dimensions without taking $\lambda_m \sim b_m$. Therefore, we will assume that the warping in the small compact dimensions is no larger than k and does not affect the magnitude of our limits. This choice gives the most lenient limits in any event.

Since the second derivatives of the warp factor vanish in the Randall-Sundrum scenario, we get the limits

$$\frac{|\lambda_m|}{M_5} < \left(\frac{M_5}{k}\right)^3 \epsilon\,, \qquad \frac{|\lambda_{mn}|}{M_5^2} < \left(\frac{M_5}{k}\right)^2 \epsilon\,. \tag{4.3}$$

Putting in specific numbers for k and ϵ, we have

$$\frac{|\lambda_m|}{M_5} < 10^{-13}, 10^{-19}\,, \qquad \frac{|\lambda_{mn}|}{M_5^2} < 10^{-14}, 10^{-20}\,. \tag{4.4}$$

These numbers suggest a type of hierarchy problem; perhaps the asymmetric warping is caused by a black hole or other gravitating object very far from our brane in the y direction. This is the usual radius stabilization problem.

Let us now relate the limits (4.3), (4.4) to the known limits on the MAV for gravitons. Specifically, we will ask in what region of parameter space do our results improve the limit $|c_g - 1| < 10^{-6}$. To do so, we must relate the Lorentz violating function Λ to the speed of gravitational wave propagation. With a global solution, we could do this by finding the higher dimensional zero-mode of the graviton, but we will need to make do with perturbation theory since we have only a solution expanded around $y^m = 0$. Our ansatz is to ignore the 5 small dimensions and use the Randall-Sundrum zero-mode as the graviton wavefunction in the warped dimension. We use this wavefunction to get an expectation value for the speed of gravity, e^{Λ}.

For perturbations around the one-brane model, we find that

$$c_g - 1 \simeq \frac{\lambda_m}{M_5}\frac{M_5}{k}\,, \qquad \frac{\lambda_{mn}}{M_5^2}\left(\frac{M_5}{k}\right)^2 . \tag{4.5}$$

Depending on which limit we use for the standard model MAVs, these give us limits better than $|c_g - 1| < 10^{-6}$ when $M_5/k < 10^{2.5}, 10^4$ for either λ_m or λ_{mn}. The two-brane model is slightly more complicated because the expectation value depends on the position of the regulator brane, y_R. However, to solve the hierarchy problem, typically $ky_R \simeq 10^2$. Then we can approximate the graviton speed by taking (4.5) and replacing $k \to 1/y_R \simeq 10^{-2}k$. This gives, for λ_m, an improvement over $|c_g - 1| < 10^{-6}$ when $M_5/k < 10^2, 10^{3.5}$ and, for λ_{mn}, an improvement when $M_5/k < 10^{1.5}, 10^3$. Since we expect $k \simeq M_5$, this covers almost all of the expected parameter space. We should also note that there is a one-sided bound of $c_g - 1 > -10^{-15}$ from gravitational Čerenkov radiation [35]. Our bounds can compete with or even improve on this more stringent bound (but without sign) when $k \simeq M_5$.

We note briefly that these bounds compare well to the field theoretic loop bounds of [10]. When using the same bounds on standard model MAVs as [10], we find an improvement on limits for c_g in precisely the region of parameter space that we naturally expect to occur. This seems to have wider applicability than bounds from loops, which are effective when the fundamental scale of the higher dimensions is itself small. In warped compactifications, this may not be the case.

4.2 Comments on cosmological constant self-tuning

Now we will comment on the application of our results to models of asymmetric warping used to solve the cosmological constant problem. As is well known, the size of the cosmological constant is as yet unexplained by theory. The review [36] considers many different possible resolutions of the cosmological constant problem, including the possibility of "tuning" by a scalar field that reduces the effect of the vacuum energy density. The conclusion is that such a method does not work in models with standard 4D physics. Nonetheless, self-tuning has enjoyed a revival of interest with the discovery by [37, 38] that vacuum energy density on a brane can be translated into curvature of a bulk scalar. Because of singularities in the bulk, however, these models require fine-tuning to reproduce 4D gravity and therefore do not excape the no-go theorem stated above [39].

Seemingly, the way around this difficulty is to place the singularity behind an event horizon; additionally, the spacetime metric can be written in a 5D Schwarzschild-like form

$$ds^2 = -h(r)dt^2 + (kr)^2 d\vec{x}^2 + h^{-1}(r)dr^2 \,, \tag{4.6}$$

where k is the same as in the Randall-Sundrum case (for $h(r) = (kr)^2$, this is just Randall-Sundrum in different coordinates). These types of solutions with a charged black hole background were first studied in [8, 12, 13, 40] and generalized in [41, 42].

Even though the full metric does not have SO$(3,1)$ symmetry, [8, 12, 13, 40] argue that these asymmetrically warped models evade limits on standard model Lorentz violation because brane fields feel an SO$(3,1)$ symmetry as long as the brane stays at a fixed position. The only effect at the field theory tree level would be to alter the speed of gravitons. We have seen that Lorentz violation can be communicated to the standard model both by loops [10] and by α' corrections in string theory as in section 3. It is natural to ask whether the limits we derived above are stringent enough to rule out these models.

The difficulty is that the warp factors at the brane are determined entirely by the energy density on the brane through jump conditions if we work only in 5D [43]. (See [43] for some of the consequences this has for cosmology.) Therefore, we find that the warp factors for Riemann normal coordinate y are given by

$$b_y = \frac{\rho}{3M_5^3} \,, \quad \lambda_y = -\frac{\rho}{M_5^3}(1+\omega) \,, \quad b_{yy} = -\frac{\rho^2}{6M_5^6}(1+\omega) \,, \quad \lambda_{yy} = 24k^2 + \frac{\rho^2}{54M_5^6}(7+60\omega) \,, \tag{4.7}$$

where ρ is the brane energy density and the brane equation of state is $P = \omega\rho$. To avoid a naked singularity in the bulk, $\omega < -1$ [12]. Ignoring this exotic equation of state and any other cosmological difficulties, our bounds (4.1) become

$$\begin{aligned}
\alpha'^2 b_y^3 \lambda_y &\simeq \left(\frac{\rho}{M_5^4}\right)^4 (1+\omega) < \epsilon \\
\alpha'^2 b_{yy} b_y \lambda_y &\simeq \left(\frac{\rho}{M_5^4}\right)^4 (1+\omega)^2 < \epsilon \\
\alpha'^2 b_y^2 \lambda_{yy} &\simeq \left(\frac{k}{M_5}\right)^2 \left(\frac{\rho}{M_5^4}\right)^2 < \epsilon \\
\alpha'^2 b_{yy} \lambda_{yy} &\simeq \left(\frac{k}{M_5}\right)^2 \left(\frac{\rho}{M_5^4}\right)^2 (1+\omega) < \epsilon \,.
\end{aligned} \tag{4.8}$$

assuming that k/M_5 is larger than ρ/M_5^4, which seems reasonable if only because of power counting. The best bound is the third one, since it avoids ambiguity due to the unspecified value of ω. Taking values for k/M_5 as before, we find $\rho/M_5^4 < 10^{-7}, 10^{-10}$. So we cannot use asymmetric warping to tune away 5D Planck scale energy densities; some other mechanism must explain why the brane energy density is at least a few orders of magnitude smaller than M_5. This somewhat lessens the appeal of the self-tuning mechanism.

It is worth noting that the constraints should be a bit tighter if we note that the warping B should not be determined solely by the brane energy density when the setup is embedded into more than 5D (because the jump conditions are no longer boundary conditions). If we take, as seems likely in a 10D scenario, $b_y \simeq k$, then we get

$$\left(\frac{k}{M_5}\right)^3 \frac{\rho}{M_5^4}(1+\omega) < \epsilon \,, \qquad \left(\frac{k}{M_5}\right)^4 < \epsilon \,. \tag{4.9}$$

If we use $k/M_5 \simeq 1/10$ as before, we find $\rho/M_5^4(1+\omega) < 10^{-13}, 10^{-19}$ as in section 4.1. However, the second relation gives $k/M_5 < 10^{-4}, 10^{-5.4}$, which is starting to reintroduce a hierarchy problem (this time in the bulk). If we accept this limit on the symmetric part of the warping, then eq. (4.9 gives

$$\frac{\rho}{M_5^4}(1+\omega) < \epsilon^{1/4} = 10^{-4}, 10^{-5.4} \,. \tag{4.10}$$

This is actually weaker (even if $1+\omega$ is order unity) than eq. (4.8), but it does use a value of the bulk energy density that may be undesirably small.

5. Discussion

We have seen how the α' expansion of string theory can communicate symmetry breaking effects between different sectors of a compactification model — in this case, Lorentz violation is transmitted from gravitation to the standard model particles. To our knowledge, this is the first use of α' corrections for such a purpose (but do note that bounds on noncommutativity and Lorentz violation in string theory have been studied [44]). We then used the Lorentz violating standard model propagators to put limits on asymmetric warping in string compactifications. These limits, in our opinion, reduce the appeal of asymmetrically warped models of cosmological constant self-tuning because they require some other mechanism to reduce the brane energy density to acceptable values of Lorentz violation. We should note again that our calculations are not complete even to $\mathcal{O}(\alpha')^2$ because the additional terms in the action due to the background supergravity field strengths are unknown; however, they are very generally applicable because they only depend on the warp factors. In addition, our limits seem somewhat stronger than the bounds derived from field theoretic loops [10]; they improve on previous limits for the speed of gravity in the interesting region of parameter space. We should also note that the limits from the α' expansion are good even if we use a relatively weak bound for standard model Lorentz violation. The key parameter in our limits is the ratio of the Lorentz invariant warp factor derivative to the fundamental Planck scale, as opposed to the Planck scale itself for loops. From naturalness considerations, we expect that the ratio will be near unity.

To close, we comment that α' corrections to the D-brane action seem very applicable to braneworld models. They may be useful in correcting the backgrounds used in the models, just as the α' corrections for the bulk action have been (see, for example, [45]).[6] They could also be used, as in this paper, to discuss the communication of symmetry breaking between brane and bulk fields. As one particular example, string compactifications with 3-form fluxes and a no-scale supergravity interpretation, which have been of great interest recently, exhibit sequestering of supersymmetry breaking [47]. That is, the brane fields do not feel the bulk breaking of supersymmetry at tree level. Although a better knowledge of α' corrections involving bulk field strengths and worldvolume fermions will be needed, it would be interesting to carry out an explicit calculation of supersymmetry breaking on the brane due to the 3-forms. As we have seen in the case of SO$(3,1)$ breaking, the α' corrections can be just as important as loop corrections. They are also somewhat easier to compute, since the known corrections can be evaluated using only differentiation and algebra and do not require justifying a regularization procedure for perturbative gravity. We hope, therefore, that we have introduced a new technology for the investigation of braneworld compactifications of string theory.

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A. Calculation of scalar kinetic term corrections

We collect here some of the details of the calculations in section 3.2, specifically the Lorentz violating scalar kinetic terms of 3.2.2. For notational convenience, we will call all spacetime coordinates X, whether they are the compact space coordinates y^m or not.

[6] Also, in Lorentz violating models, nonrenormalizable terms may be needed to maintain causality [46].

We start by specifying the tangent to the D-brane, which we take as $\partial_\alpha X^\mu = \delta^\mu_\alpha$ with perturbatively small $\partial_\alpha X^m$. Then the normal bundle vielbein satisfies

$$\partial_\alpha X^M \xi^{\hat{a}}_M = 0 \qquad \text{or} \qquad \xi^{\hat{a}}_\mu = -\delta^\alpha_\mu \partial_\alpha X^m \xi^{\hat{a}}_m \tag{A.1}$$

which has solution to second order

$$\xi^{\hat{a}}_\mu = -\delta^\alpha_\mu \partial_\alpha X^m \tilde{e}^{\hat{m}}_m \delta^{\hat{a}}_{\hat{m}}\,, \qquad \xi^{\hat{a}}_m = \delta^{\hat{a}}_{\hat{m}} \tilde{e}^{\hat{m}}_m - \frac{1}{2} g^{\mu\nu} \delta^\alpha_\mu \delta^\beta_\nu \partial_\alpha X^p \partial_\beta X^n \hat{e}^{\hat{n}}_n \delta^{\hat{a}}_{\hat{n}} \tilde{g}_{mp}\,. \tag{A.2}$$

Here $\tilde{e}^{\hat{m}}_m$ is the vielbein of $\tilde{g}_{mn}$ and is trivial at the position of the brane. Since its expansion would just give powers of X^m and not derivatives, we will ignore it from now on. Also, for notational convenience, since the normal bundle indices $\hat{a}$ always enter through Kronecker deltas with compact space indices, we will abuse notation slightly and replace $\hat{a}$ with m in future formulae.

The nonvanishing Christoffel symbols are

$$\Gamma^0_{0m} = \frac{1}{2}\partial_m A\,, \qquad \Gamma^m_{00} = \frac{1}{2}\tilde{g}^{mn} e^A \partial_n A\,, \qquad \Gamma^i_{jm} = \frac{1}{2}\partial_m B \delta^i_j\,, \qquad \Gamma^m_{ij} = -\frac{1}{2}\tilde{g}^{mn} e^B \partial_n B \delta_{ij} \tag{A.3}$$

on the spacetime and

$$\begin{aligned}
(\Gamma_T)^0_{00} &= \frac{1}{2}\partial_0 X^m \partial_m A\,, \qquad (\Gamma_T)^0_{0i} = \frac{1}{2}\partial_i X^m \partial_m A\,, \qquad (\Gamma_T)^i_{00} = \frac{1}{2} e^{A-B} \delta^{ij} \partial_j X^m \partial_m A \\
(\Gamma_T)^0_{ij} &= \frac{1}{2} e^{B-A} \delta_{ij} \partial_0 X^m \partial_m B\,, \qquad (\Gamma_T)^i_{0j} = \frac{1}{2}\delta^i_j \partial_0 X^m \partial_m B \\
(\Gamma_T)^i_{jk} &= \frac{1}{2}\left(\delta^i_k \partial_j X^m + \delta^i_j \partial_k X^m - \delta_{jk}\delta^{il}\partial_l X^m\right)\partial_m B
\end{aligned} \tag{A.4}$$

on the tangent space. This is just the Christoffel symbol of the pulled-back metric $g_{\alpha\beta}$. Plugging into eq. (3.8), we can very easily get eq. (3.12) for the extrinsic curvature.

Then, from eq. (3.9), the tangent Riemann components out to $\mathcal{O}(\partial X^m)^2$ are

$$\begin{aligned}
(R_T)_{0i0j} &= -\frac{1}{4} a^m b_m \delta_{ij} + \frac{1}{2} a_m b_n \delta_{ij} \partial_0 X^m \partial_0 X^n - \frac{1}{2} a_{(m} b_{n)} \partial_i X^m \partial_j X^n - \\
&\quad -\frac{1}{2}\left(a_{mn} + \frac{3}{2} a_m a_n\right) \partial_i X^m \partial_j X^n + \frac{1}{2}\left(b_{mn} + \frac{1}{2} b_m b_n\right) \delta_{ij} \partial_0 X^m \partial_0 X^n \\
(R_T)_{ijkl} &= \frac{1}{4} b^2 \left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right) + \frac{1}{2}\left(b_{mn} + \frac{3}{2} b_m b_n\right) \times \\
&\quad \times \left(\partial_i X^m \partial_k X^n \delta_{jl} + \partial_j X^m \partial_l X^n \delta_{ik} - \partial_i X^m \partial_l X^n \delta_{jk} - \partial_j X^m \partial_k X^n \delta_{il}\right).
\end{aligned} \tag{A.5}$$

The other components are only $\mathcal{O}(\partial X^m)^2$, so they square to $\mathcal{O}(\partial X^m)^4$. When squaring, we also need to take into account the second order part of the inverse metric pull-back. Note that the only terms involving Λ come from $(R_T)_{0i0j}$. To get $(R_T)_{\alpha\beta}$ we just contract the Riemann tensor, being careful of second order terms in the metric.

From eq. (3.10), the normal bundle Riemann tensor is always second order in ∂X^m, so it does not contribute to the kinetic terms. That leaves just $\bar{R}^{mn}$. We end up with

$$\begin{aligned}\bar{R}^{mn} = & -\frac{1}{2}\left(a^{mn}+a^m a^n\right)-\frac{3}{2}b^{mn}+\tilde{R}_{pmnq}\partial_\mu X^p\partial^\mu X^q-\left(a_p{}^{(m}+\frac{3}{2}a_p a^{(m}\right)\partial_0 X^{n)}\partial_0 X^p- \\ & -\frac{1}{2}\left(b_p{}^{(m}-\frac{3}{2}b_p b^{(m}\right)\vec{\partial}X^m\cdot\vec{\partial}X^n-\frac{1}{2}\left(a^{mn}+\frac{5}{2}a^m a^n\right)\delta_{pq}\partial_0 X^p\partial_0 X^q+ \\ & +\frac{1}{2}\left(b^{mn}-\frac{1}{2}b^m b^n\right)\delta_{pq}\vec{\partial}X^p\cdot\vec{\partial}X^q\,. \end{aligned} \tag{A.6}$$

Here $\tilde{R}_{mnpq}$ is the Riemann tensor of $\tilde{g}_{mn}$; note that it enters in a Lorentz invariant fashion. Getting the action (3.13) is just a matter of squaring.

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